1) $$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} = -g \frac{\partial h}{\partial x}$$

$$x^* = \frac{x}{L} \qquad v_x^* = \frac{v_x}{v_\infty} \qquad h^* = \frac{h}{L} \qquad t^* = \frac{t\, v_\infty}{L}$$

$$\frac{\partial v_x}{\partial t} = \frac{\partial (v_x^* v_\infty)}{\partial t^*} \frac{\partial t^*}{\partial t} = \frac{v_\infty^2}{L} \frac{\partial v_x^*}{\partial t^*}$$

$$\frac{\partial v_x}{\partial x} = \frac{\partial (v_x^* v_\infty)}{\partial x^*} \frac{\partial x^*}{\partial x} = \frac{v_\infty}{L} \frac{\partial v_x^*}{\partial x}$$

$$\frac{\partial h}{\partial x} = \frac{\partial (h^* L)}{\partial x^*} \frac{\partial x^*}{\partial x} = \frac{\partial h^*}{\partial x^*}$$

$$\frac{v_\infty^2}{L} \frac{\partial v_x^*}{\partial t^*} + \frac{v_\infty^2}{L} \frac{v_x^* \partial v_x^*}{\partial x^*} = -g \frac{\partial h^*}{\partial x^*}$$

$$\frac{\partial v_x^*}{\partial t} + v_x^* \frac{\partial v_x^*}{\partial x^*} = - \boxed{\frac{gL}{v_\infty^2}} \frac{\partial h^*}{\partial x^*}$$

This is the inverse of the Froude number squared.

2) $\omega = F(b, h, V, \rho, K$

reference variables chosen: ρ V h

$\Pi_1 = \frac{b}{h}$ $\Pi_2 = \frac{\omega h}{V}$ $\Pi_3 = \frac{K}{\rho V^2 h}$

$\therefore \frac{\omega h}{V} = g\left(\frac{b}{h}, \frac{K}{\rho V^2 h}\right)$

3) $F = f_1(D, \omega, V, \mu, \rho)$

$T = f_2(D, \omega, V, \mu, \rho)$

repeated variables $\rho V D$

$\Pi_1 = \frac{\rho V D}{\mu} = Re$ $\quad \Pi_2 = \frac{\omega D}{V} = St$ (strouhal #)

$\Pi_3 = \frac{F}{\rho V^2 D^2}$ $\quad \Pi_4 = \frac{T}{\rho V^2 D^3}$

a) $Re_1 = Re_2$ $\quad \frac{\cancel{\rho_1} V_1 D_1}{\cancel{\mu_1}} = \frac{\cancel{\rho_2} V_2 D_2}{\cancel{\mu_2}}$

$V_2 = V_1 \frac{D_1}{D_2} = (200 Km/h) \frac{(0.5m)}{3m}$

$= 33 \ Km/h$

$\frac{\omega_1 D_1}{V_1} = \frac{\omega_2 D_2}{V_2}$

$$\omega_2 = \omega_1 \frac{D_1}{D_2} \frac{V_2}{V_1} = 2000 rpm \left(\frac{0.5 m}{3 m}\right)\left(\frac{33 km/h}{200 km/h}\right)$$

$$= 55 rpm$$

b) $$\frac{F_1}{\rho_1 V_1^2 D_1^2} = \frac{F_2}{\rho_2 V_2^2 D_2^2}$$

$$F_2 = F_1 \frac{(\cancel{\rho_2} V_2^2 D_2^2)}{(\cancel{\rho_1} V_1^2 D_1^2)}$$

$$= 92 N \left(\frac{33 km/h}{200 km/h}\right)^2 \left(\frac{3 m}{0.5 m}\right)^2 = \boxed{92 N}$$

$$\frac{T_1}{\rho_1 V_1^2 D_1^3} = \frac{T_2}{\rho_2 V_2^2 D_2^3}$$

$$T_2 = 12 Nm \left(\frac{\cancel{\rho_2} V_2^2 D_2^3}{\cancel{\rho_1} V_1^2 D_1^3}\right) = \boxed{72 Nm}$$

4) $F_L = f(L, V, \rho, \mu, c)$ use: ρ, V, L as repeated

a) $\Pi_1 = \frac{F_L}{\rho V^2 L^2}$ $\Pi_2 = \frac{\rho V L}{\mu} = Re$ $\Pi_3 = \frac{V}{c} = Ma$

b) Real: $Re = \frac{(0.365\ kg/m^3)(250\ m/s)(30\ m)}{(1.4\times10^{-5}\ Pa\,s)} = 1.96\times10^8$

$$Ma = \frac{V}{c} = \frac{250\ m/s}{295\ m/s} = 0.847$$

Air tunnel: $Ma = 0.847 = \frac{V}{c}$

$$V = 0.847\,(340\ m/s) = 288\ m/s$$

$Re = 1.96\times10^8 \Rightarrow L = \frac{(1.96\times10^8)(1.8\times10^{-5}\ Pa\,s)}{(1.225\ kg/m^3)(288\ m/s)}$

$$\boxed{L = 10\ m}$$

Doesn't fit in tunnel!

Nitrogen tunnel:

$$Ma = 0.847 = V/c$$

$$V = 0.847\,(274\,m/s) = 232\,m/s$$

$$Re = 1.96 \times 10^8 \Rightarrow L = \frac{(1.96\times10^8)(1.2\times10^{-5}\,Pa\,s)}{(7.482\,kg/m^3)(232\,m/s)}$$

$$\boxed{= 1.35\,m} \quad \text{Fits}$$

c) $$\left(\frac{F_L}{\rho V^2 L^2}\right)_{real} = \left(\frac{F_L}{\rho V^2 L^2}\right)_{model}$$

$$\frac{(F_L)_{real}}{(F_L)_{model}} = \frac{(\rho V^2 L^2)_{real}}{(\rho V^2 L^2)_{model}}$$

Air: $$\frac{(F_L)_{real}}{(F_L)_{model}} = \boxed{2.02}$$

N_2: $$\frac{(F_L)_{real}}{(F_L)_{model}} = \boxed{27.97}$$